Memory effects in a turbulent dynamo: Generation and propagation of a large-scale magnetic field

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We are concerned with large-scale magnetic-field dynamo generation and propagation of magnetic fronts in turbulent electrically conducting fluids. An effective equation for the large-scale magnetic field is developed here that takes into account the finite correlation times of the turbulent flow. This equation involves the memory integrals corresponding to the dynamo source term describing the alpha effect and turbulent transport of magnetic field. We find that the memory effects can drastically change the dynamo growth rate, in particular, nonlocal turbulent transport might increase the growth rate several times compared to the conventional gradient transport expression. Moreover, the integral turbulent transport term leads to a large decrease of the speed of magnetic front propagation.

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I. INTRODUCTION

The problem of the generation and propagation of a magnetic field in turbulent electrically conducting fluids is of fundamental importance due to various applications in plasma physics, astrophysics, geophysics, etc. [1–5]. It has attracted much attention since the late 1960s when it was realized that the conditions for the occurrence of a large-scale magnetic field can be found by applying a simple technique involving the mean helicity of the turbulence [3]. This technique is based on the effective macroscopic equation governing large-scale magnetic field **B**. The standard form of the mean-field dynamo equation is

$$\frac{\partial \mathbf{B}}{\partial t}=\mathrm{curl}(\alpha\mathbf{B})+\beta\Delta\mathbf{B}+\mathrm{curl}(\mathbf{u}\times\mathbf{B}), \tag{1}$$

where **u** is the mean velocity field, α is the helicity, and β is the turbulent diffusivity. Equation (1) is a common starting point for analyzing the generation of the large-scale magnetic field [1–8]. It has been also used for the analysis of the propagation of magnetic fronts in spiral galaxies [9–11].

The main disadvantage of Eq. (1) is that it has been derived for turbulent flow involving only two separated length scales for the velocity field—the integral length scale and the small turbulent scale [3]. It is clear that the assumption of two separated scales is rather unrealistic for fully developed turbulent flow, which involves a continuous range of spatial and temporal scales [6]. Thus, the purely local Eq. (1) is applicable only under the assumption of a clear-cut separation between ''macroscopic'' behavior of the averaged magnetic field and the turbulent fluctuations at the ''microscopic'' level.

It should be noted that Eq. (1) is similar to convection-diffusion-reaction equations [12,13]. In fact, it can be reduced to the famous Fisher-Kolmogorov-Petrovskii-Piskunov (FKPP) equation [5,9], which has become a basic mathematical tool in the theory of propagating fronts travelling into the unstable state of the reaction-diffusion systems. There has been an increased interest in this topic, because of the large number of physical, chemical, and biological problems that can be treated in terms of the FKPP equation (see, for example, [12–14]). Recently, there has been tremendous activity to extend this analysis by introducing a more realistic description of the transport processes. The main motivation for this is that the diffusion approximation for transport admits an infinite speed of propagation. Due to this nonphysical property of the diffusion solution, the FKPP equation yields an overestimation of the propagation speed of traveling fronts [14–16]. We expect that a similar situation might take place in magnetohydrodynamics. The mean-field dynamo Eq. (1) also admits an infinite speed of magnetic-field propagation. It is clearly a nonphysical property, because the speed of the magnetic-field propagation cannot exceed the velocity of the largest eddy of turbulent flow. The origin of this contradiction lies in the δ-correlated-in-time approximations for the turbulent velocity field (see below). In reality, these correlations have finite times of relaxation, and what is more, these might be of the same order as the characteristic times for the growth rate of the large-scale magnetic field since the physical origin of these correlations and the magnetic field generation are the same, namely, the turbulent fluctuations.

II. MEAN-FIELD DYNAMO EQUATION WITH MEMORY

It is the aim of this paper to extend mean-field dynamo Eq. (1) to the case when finite time correlations of turbulent flow are taken into account, and find out how the nonlocal in time effect might influence the critical conditions for the generation of magnetic field and its spatial propagation. Here, we do not consider feedback of magnetic fluctuations on the large-scale turbulent velocity field. The latter nonlin-

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ear dynamo problem is extremely difficult to deal with and little is known about the nonlinear saturation regime.

The most general phenomenological formulation of the dynamo problem that is considered here is represented by the equations for the average magnetic field **B** [3]:

$$\frac{\partial \mathbf{B}}{\partial t}=\operatorname{curl}\mathcal{E}+\operatorname{curl}(\mathbf{u}\times\mathbf{B}),\quad \mathcal{E}=\langle\mathbf{u}'\times\mathbf{B}'\rangle, \tag{2}$$

where primes denote the turbulent fluctuations and the angular brackets denote an ensemble averaging over the turbulent pulsations. The main closure problem here is to express turbulent electromotive force $\mathcal{E}$ in terms of average field **B**. The classical expression leading to Eq. (1) is

$$\mathcal{E}=\alpha\,\mathbf{B}-\beta\operatorname{curl}\mathbf{B},$$

where α and β are the statistical characteristics of turbulent velocity $\mathbf{u}'$. However, under the assumptions of infinite conductivity, the electromotive force $\mathcal{E}$ can be written as [3]:

$$\mathcal{E}(\mathbf{x},t)=-\frac{1}{3}\int_{-\infty}^{t}\langle\mathbf{u}'(\mathbf{x},t)\cdot\operatorname{curl}\mathbf{u}'(\mathbf{x},s)\rangle\mathbf{B}(\mathbf{x},s)ds$$
$$-\frac{1}{3}\int_{-\infty}^{t}\langle\mathbf{u}'(\mathbf{x},t)\cdot\mathbf{u}'(\mathbf{x},s)\rangle\operatorname{curl}\mathbf{B}(\mathbf{x},s)ds. \tag{3}$$

The local mean-field Eq. (1) can be derived from Eqs. (2) and (3) under the assumptions that the correlations appearing in Eq. (3) are approximated by the delta functions in time:

$$\langle\mathbf{u}'(\mathbf{x},t)\cdot\operatorname{curl}\mathbf{u}'(\mathbf{x},s)\rangle=-3\alpha\,\delta(t-s),$$
$$\langle\mathbf{u}'(\mathbf{x},t)\cdot\mathbf{u}'(\mathbf{x},s)\rangle=3\beta\,\delta(t-s).$$

However, as was mentioned in Ref. [4] (p. 136) "... the assumption of instantaneous correlations seems to be a serious restriction to the theory ...," since in real turbulence the characteristic times of these correlations are finite. It follows from Eq. (3) that in the case of finite correlation times the electromotive force should contain the integrals over the history of **B** and curl **B**. In this paper we suggest the following general form for electromotive force $\mathcal{E}$ in the limit of infinite conductivity:

$$\mathcal{E}=\alpha(\mathbf{x})\int_{-\infty}^{t}G_{\alpha}\left(\frac{t-s}{\tau_{\alpha}}\right)\mathbf{B}(\mathbf{x},s)ds$$
$$-\beta(\mathbf{x})\int_{-\infty}^{t}G_{\beta}\left(\frac{t-s}{\tau_{\beta}}\right)\operatorname{curl}\mathbf{B}(\mathbf{x},s)ds, \tag{4}$$

where $G_{\alpha}(y)$ and $G_{\beta}(y)$ are positive, decreasing functions that tend to zero as $y\to\infty$. Parameters τ_{α} and τ_{β} control the time correlations of the random velocity field at a fixed space position. Formula (4) represents the generalization of the classical relation $\mathcal{E}=\alpha\mathbf{B}-\beta\operatorname{curl}\mathbf{B}$ for the case when the correlation memory of turbulent flow is taken into account. By inserting Eq. (4) into Eq. (2) one can get the nonlocal in time mean-field dynamo equation

$$\frac{\partial \mathbf{B}}{\partial t}=\int_{-\infty}^{t}G_{\alpha}\left(\frac{t-s}{\tau_{\alpha}}\right)\operatorname{curl}[\alpha\mathbf{B}(\mathbf{x},s)]ds$$
$$-\int_{-\infty}^{t}G_{\beta}\left(\frac{t-s}{\tau_{\beta}}\right)\operatorname{curl}[\beta\operatorname{curl}\mathbf{B}(\mathbf{x},s)]ds+\operatorname{curl}(\mathbf{u}\times\mathbf{B}). \tag{5}$$

It should be noted that Eq. (5) is valid only in the limit of infinitely large conductivity. This case is of specific interest for magnetohydrodynamics of plasma and many problems of astrophysics and geophysics [3]. Many different constitutive models might arise from different choices of $G_{\alpha}(y)$ and $G_{\beta}(y)$. Equation (1) can be considered as a limiting case of Eq. (5) when $G_{\alpha,\beta}(t-s)=\delta(t-s)$ $(\tau_{\alpha,\beta}\to 0)$. Transport memory kernel G_{β} ensures the finite velocity of propagation of a magnetic field, which is determined by the rate of turbulent pulsations [6]. Relaxation time τ_{β} can be determined from the root-mean-square velocity $u_{\mathrm{rms}}=\sqrt{\beta/\tau_{\beta}}$ of turbulent fluctuations. Integral kernel G_{α} is used to express the fact that the dynamo growth of a magnetic field at a time t in the local vicinity of spatial point **x** is determined by the past values of curl **B**. It should be noted that some aspects of the influence of correlation memory on the intensity of the α effect have been studied by Cattaneo and co-authors [5]. It has been shown that "... the effect of a large-scale magnetic field is to induce a long-term memory in the field of turbulence" and this leads to the feedback influence resulting in partial suppression of the α generation of a magnetic field.

The implications of our results for dynamo theory are twofold. First, the nonlocality in time greatly influences the dynamo growth rate of a magnetic field, however, the generation conditions are not sensitive to the addition of the memory effect due to nonzero time correlations: they are robust to the addition of nonlocal terms for mean-field dynamo Eq. (5). Second, the memory effects introduce drastic changes to the front dynamics of a magnetic field.

III. GROWTH RATE FOR THE MEAN FIELD DISK DYNAMO

In what follows, we study the influence of the memory effects on the dynamo generation and propagation by using an important example of a thin turbulent slab of thickness $2h$ and radius $R(R\gg h)$, which rotates with angular velocity $\omega(r)$. This is a standard model for disk-like galaxies. We neglect the effects of compressibility, diamagnetism, and deviations from the axial symmetry. We use the standard approximation of constant turbulent diffusion coefficient β [4,5,9–11]. We restrict our analysis to the kinematical aspects of the problem, neglecting the influence of the magnetic field on the turbulent flow, i.e., the dependencies of α, β, τ_{α}, and τ_{β} on magnetic field **B**. With these simplifications, the equations for the components of mean axisymmetric magnetic field $B_r(t,r,z)$ and $B_{\varphi}(t,r,z)$ in the polar cylindrical coordinates (r,φ,z) with the z axis coincident with the rotation axis follow from Eq. (5):

$$\frac{\partial B_r}{\partial t}=-\int_{-\infty}^{t}G_\alpha\left(\frac{t-s}{\tau_\alpha}\right)\frac{\partial}{\partial z}(\alpha B_\varphi)ds+\beta\int_{-\infty}^{t}G_\beta\left(\frac{t-s}{\tau_\beta}\right)\times\left\{\frac{\partial^2 B_r}{\partial z^2}+\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(rB_r)\right]\right\}ds, \tag{6}$$

$$\frac{\partial B_\varphi}{\partial t}=g\,B_r+\int_{-\infty}^{t}G_\alpha\left(\frac{t-s}{\tau_\alpha}\right)\frac{\partial}{\partial z}(\alpha B_r)ds+\beta\int_{-\infty}^{t}G_\beta\left(\frac{t-s}{\tau_\beta}\right)\times\left\{\frac{\partial^2 B_\varphi}{\partial z^2}+\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(rB_\varphi)\right]\right\}ds. \tag{7}$$

Here, $g=rd\omega/dr$ is the measure of differential rotation, and we are interested only in the $B_r(t,r,z)$ and $B_\varphi(t,r,z)$ components of a magnetic field, because $B_z/B_{r,\varphi}=O(h/R)$ [4,5]. These components obey the vacuum boundary conditions on the thin disk surfaces [4,5]:

$$B_{r,\varphi}(t,r,-h)=0,\quad B_{r,\varphi}(t,r,h)=0.$$

The main goal of our analysis is to find the local growth rate of the magnetic field and its spatial propagation, taking into account the memory effects. First, consider magnetic-field generation in the usual way [4,5,9,10], by neglecting the radial derivatives in Eqs. (6) and (7). We represent the components of the magnetic field **B** as follows:

$$B_r(t,z)=b_r(z)\exp(\gamma t),\quad B_\varphi(t,z)=b_\varphi(z)\exp(\gamma t),$$

where $b_r(z)$ and $b_r(z)$ have to be found from the eigenvalue problem

$$\left(\tilde{\gamma}+\frac{\partial^2}{\partial z^2}\right)b_r=-\tilde{R}_\alpha\frac{\partial(\alpha b_\varphi)}{\partial z},$$

$$\left(\tilde{\gamma}+\frac{\partial^2}{\partial z^2}\right)b_\varphi=\tilde{R}_w b_r+\tilde{R}_\alpha\frac{\partial(\alpha b_r)}{\partial z},$$

$$b_r(1)=b_r(-1)=b_\varphi(1)=b_\varphi(-1)=0, \tag{8}$$

where

$$\tilde{\gamma}=\gamma/f_\beta(\gamma T_\beta),\quad \tilde{R}_w=R_w/f_\beta(\gamma T_\beta),$$

$$\tilde{R}_\alpha=R_\alpha f_\alpha(\gamma T_\alpha/R_\alpha)/f_\beta(\gamma T_\beta),$$

$$f_{\alpha,\beta}(\gamma\theta_{\alpha,\beta})=\int_0^\infty G_{\alpha,\beta}(\xi/\theta_{\alpha,\beta})\exp(-\gamma\xi)d\xi,$$

$$\theta_\alpha=T_\alpha/R_\alpha,\quad \theta_\beta=T_\beta.$$

Here, we use the dimensionless variables $z\to z/h$, $t\to\beta t/h^2$, $\alpha\to\alpha_0\alpha(z)$, and dimensional parameters $T_\alpha=\alpha_0\tau_\alpha/h$, $T_\beta=\beta\tau_\beta/h^2$, $R_\alpha=\alpha_0 h/\beta$, $R_w=gh^2/\beta$. Parameter γ describes the growth rate of the magnetic field. The eigenvalue problem (8) reduces to the well-known form [4,5,9,10] in the case when $T_{\alpha,\beta}=0$. Otherwise, eigenvalue γ becomes a function of these correlation times, and this function is determined from problem (8) using the renormalized parameters $\tilde{\gamma}$, $\tilde{R}_\alpha$, $\tilde{R}_w$.

Problem (8) coincides with the generation equations in the local mean-field dynamo theory [4,5,9,10]. The only difference is that renormalized parameters $\tilde{\gamma}$, $\tilde{R}_\alpha$, and $\tilde{R}_w$ are dependent on increment γ of the time growth of the magnetic field. This means that generation equations (8) are universal and do not depend on the specific choice of memory kernels $G_{\alpha,\beta}$. The main physical conclusion from this universality is that since physically meaningful memory kernels must satisfy the normalization condition $f_{\alpha,\beta}(\gamma=0)=1$, then the threshold combinations of dynamo parameters R_α and R_β at a given $\alpha(z)$, providing the critical point of instability (Re $\gamma=0$), are the same for all memory kernels. These critical parameters can be determined from the local mean-field dynamo theory. To determine growth rate γ in the generation region, eigenvalue problem (8) should be solved as $\tilde{\gamma}=\tilde{\gamma}(\tilde{R}_\alpha,\tilde{R}_w)$. This relation with the definitions of the renormalized parameters $\tilde{\gamma}$, $\tilde{R}_\alpha$, $\tilde{R}_w$ leads to a transcendental equation for γ. The specific dependence of γ on the relaxation times $T_{\alpha,\beta}$ and dynamo parameters $R_{\alpha,w}$ has to be determined by the specific forms of memory kernels $G_{\alpha,\beta}$.

Let us illustrate the general results by using the important example of $\alpha\omega$ dynamo ($R_\alpha\ll|R_w|$), which is of specific interest for astrophysical problems [3–5,9–11]. We use the exponential relaxation forms

$$G_\alpha\left(\frac{t-s}{\tau_\alpha}\right)=\frac{1}{\tau_\alpha}\exp\left(-\frac{t-s}{\tau_\alpha}\right),$$

$$G_\beta\left(\frac{t-s}{\tau_\beta}\right)=\frac{1}{\tau_\beta}\exp\left(-\frac{t-s}{\tau_\beta}\right). \tag{9}$$

The solution of the eigenvalue problem (8) depends on the form of the function $\alpha(z)$. The $\alpha\omega$ approximation requires the antisymmetric character $\alpha(z)=-\alpha(-z)$. Following the asymptotic method of the solution of eigenvalue problem (8) [4,5,10], for the model case $\alpha(z)=z$, the increment of maximal growth satisfies the algebraic equation:

$$\gamma(1+\gamma T_\beta)=-\frac{\pi^2}{4}+\sqrt{D}\frac{1+\gamma T_\beta}{\sqrt{1+\gamma T_\alpha/R_\alpha}},\quad D=-R_\alpha R_w. \tag{10}$$

Growth rate γ is shown in Fig. 1 as a function of dimensionless correlation times T_α and T_β. Clearly, the nonlocal terms corresponding to the turbulent helicity and the turbulent transport influence growth rate γ in opposing ways. The turbulent α-source nonlocality leads to a slower field growth (curve 1) due to the physical nature of the process: the integral representation of the α term in basic Eqs. (4) and (5) introduces the time memory of excitation. The nonlocality of turbulent β transport reduces the energy loss out of the disk and, hence, results in an increase of the field growth rate (curve 2). Moreover, for dynamo numbers D, close to critical value D_{cr}, the influence of the transport nonlocality becomes very significant, and growth rate γ increases drastically with

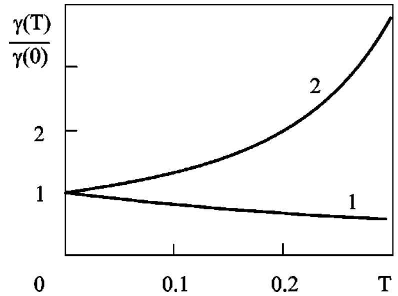

FIG. 1. Dependencies of the dimensionless growth rate γ on turbulent α-source relaxation time T_α at $T_\beta=0$ (curve 1) as well as on transport time T_β when $T_\alpha=0$ (curve 2). Memory kernels $G_{\alpha,\beta}$ are chosen to have the form of Eq. (9); the dimensionless dynamo parameters of the system are: $R_\alpha=1$ and $R_w=6.1$.

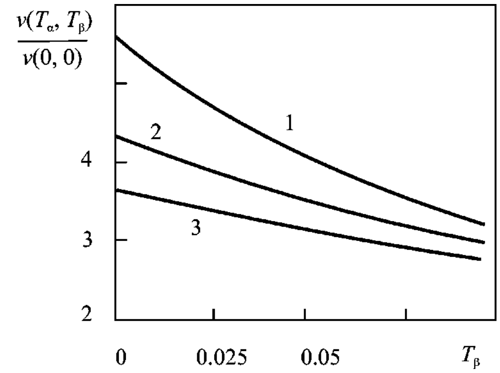

FIG. 2. Dependence of dimensionless front velocity $\tilde{v}=v(T_\alpha,T_\beta)/v(0,0)$ on relaxation time T_β at $T_\alpha=0$ (curve 1), 0.3 (curve 2), and 0.9 (curve 3). The dynamo parameters are $R_\alpha=2$ and $R_w=-40$.

T_β. It is worth noting that the critical value $D_{\rm cr}\simeq\pi^4/16$ of the dynamo number D, at which the generation commences $(\gamma=0)$, does not depend on the memory effects; this result is in agreement with the general conclusion made above.

IV. MAGNETIC FRONT PROPAGATION FOR THE DISK DYNAMO

Now we are in a position to discuss the problem of magnetic front propagation. If the dynamo excitation takes place within a certain radius (say, $r<r_0$), then the magnetic field can propagate in the form of traveling fronts [10]. According to the local dynamo theory, the speed v of a propagating magnetic front is proportional to $\sqrt{\beta\gamma}$ due to the FKPP type of mean-field Eq. (1). The problem is that in the local mean-field description of magnetic fields, turbulent diffusion induces an infinitely long Gaussian tail ahead of the magnetic front which leads to the overestimation of the propagation rate. A qualitatively different situation appears when the memory effects are taken into account. This means that speed v cannot exceed the maximum possible velocity $\sqrt{\beta/\tau_\beta}$, however large the growth rate γ is. This can be explained as follows. Let us consider the simple situation when $\mathbf{u}=0$ and $\tau_\alpha=0$, whereas $\tau_\beta\neq0$. By using the exponential expression for memory kernel G_β, the nonlocal mean-field dynamo Eq. (5) can be rewritten as a local equation involving the second derivative with respect to time

$$\tau_\beta\frac{\partial^2\mathbf{B}}{\partial t^2}+\frac{\partial}{\partial t}[\mathbf{B}-\tau_\beta\,\mathrm{curl}(\alpha\mathbf{B})]=\mathrm{curl}(\alpha\mathbf{B})+\beta\Delta\mathbf{B}.$$

It is well known that equations of such type, unlike parabolic equations, correspond to the transport phenomena with the finite maximal propagation velocity being equal to $\sqrt{\beta/\tau_\beta}$. Therefore, integral nonlocal model (5) takes into account the fact that the dynamo propagation of magnetic field is characterized by a finite maximal velocity, which is determined by the turbulent pulsations.

Now let us turn to the problem of front propagation in the case when the dynamo equation with memory (5) is considered in the thin disk approximation. Assume that the initial distribution of magnetic field satisfies: $B=B_0=\mathrm{const}$ if $r<r_0$ and $B=0$ otherwise. The main quantity of interest is the speed v at which the magnetic field propagates in the form of a self-similar wave for large values of r and t. In the large-distance limit $(r\to\infty)$ the radial Laplace operator $\partial/\partial r[1/r(\partial(rB)/\partial r)]$ in Eqs. (6) and (7) can be approximated by the second derivative $\partial^2B/\partial r^2$ and we may find the solution of these equations in the form of Fourier modes:

$$B_r(t,r,z)=b_r(z)\exp(\gamma t+ikr),\quad B_\varphi(t,r,z)=b_\varphi(z)\exp(\gamma t+ikr).$$

Substitution of these expressions into Eqs. (6) and (7) leads to the problem for eigenfunctions $b_{r,\varphi}$ [4,5], from which one can find the equation for the exponent $\gamma(k)$ as a function of wave number k. The general theory of front propagation in nonlocal reaction-transport media [15,16], based on the saddle-point method of calculation of the inverse Fourier integrals, leads to the following equations for the propagating velocity:

$$v=\frac{\gamma(p)}{p},\quad \frac{\gamma(p)}{p}=\frac{d\gamma(p)}{dp},$$

where $p=ik$. We have found that front velocity v is a monotonically decreasing function of both correlation times τ_α and τ_β. In particular, it depends weakly on τ_α (see Fig. 2), but decreases significantly (up to 2–3 times) with growing τ_β. Therefore, the use of the FKPP-like estimation for the traveling wave velocity $v\sim\sqrt{\beta\gamma}$ for systems with memory leads to significant overestimations. This general conclusion is of great importance not only for magnetic dynamo front propagation but for a wide class of excitable media.

V. SUMMARY

Basically, we have extended the classical mean-field dynamo theory to the case when the memory effects are taken into account. We have suggested the effective equation for the large-scale magnetic field that takes into account the finite correlation times of the turbulent flow. This equation involves memory integrals corresponding to the dynamo source term describing the alpha effect and turbulent transport of magnetic field. We have found that memory effects can drastically change the dynamo growth rate, in particular, the finite turbulent transport involving memory might increase the growth rate several times. We have also found that memory effects lead to the essential decrease of the speed of magnetic front propagation.

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